POSTAL BOOK PACKAGE 2026

MADE EASY PUBLICATIONS

ELECTRONICS ENGINEERING

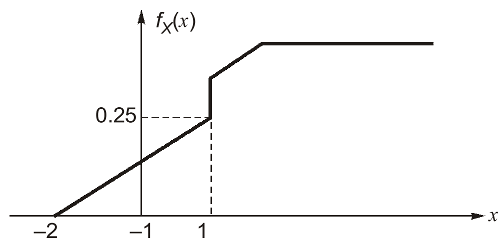
CONVENTIONAL Practice Sets

CONTENTS

COMMUNICATION SYSTEMS

1 CHAPTER Theory of Random Variable and Noise

Q.1 Define PDF and summarise its important properties. Also calculate the probability of outcome of a Random Variable (RV) X having $X \leq 1$ for the following PDF curve of RV as shown.



Solution:

Probability density function specifies the probability of a random variable taking a particular value.

The Probability Density Function (PDF) which is generally denoted by $f_X(x)$ or $P_X(x)$ or $\rho_X(x)$ is defined in terms of the Cumulative Distribution Function (CDF) $F_X(x)$ as,

$$\boxed{\text{PDF} = f_X(x) = \frac{d}{dx} F_X(x)} \qquad \text{...(i)}$$

The PDF has the following properties:

(i) $f_X(x) \geq 0$ for all x

This results from the fact that probability cannot be negative. Also, $F_X(x)$ increases monotonically, as x increases, more outcomes are included in the prob. of occurrence represented by $F_X(x)$.

(ii) Area under the PDF curve is always equal to unity.

i.e. $$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

(iii) The CDF is obtained by the result

$$\text{CDF} = \int_{-\infty}^{x} f_X(x)dx$$

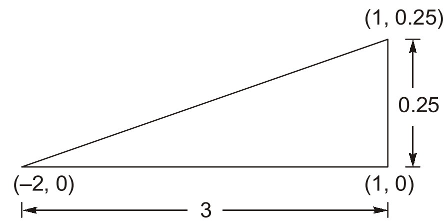
(iv) Probability of occurrence of the value of random variable between the limits of x_1 and x_2 is given by,

$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} f_X(x)dx$$

Now consider the given PDF curve, since we have to find $P(x \leq 1)$ so,

Equation for the PDF curve for $x \leq 1$ is,

$$f_X(x) = \left(\frac{1}{12}x + \frac{1}{6}\right)$$



Now, $P(x \le 1)$

$$= P(-2 < x < 1) = \int_{-2}^{1}\left(\frac{1}{12}x+\frac{1}{6}\right)dx = \left[\frac{1}{12}\cdot\frac{x^2}{2}+\frac{1}{6}x\right]_{-2}^{1} = \frac{3}{8}$$

$$\therefore \qquad P(x \le 1) = \frac{3}{8}$$

Q.2 Find the cumulative distribution function $F(x)$ corresponding to the PDF $f(x) = \dfrac{1}{\pi(1+x^2)}, -\infty < x < \infty$.

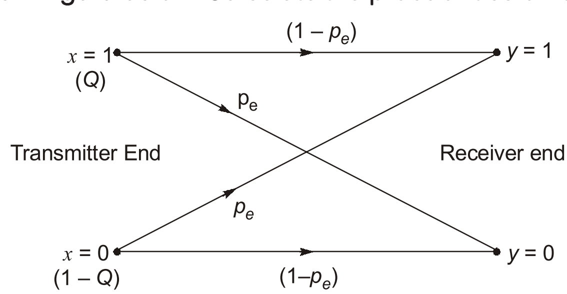
Solution:

Given $\qquad f(x) = \dfrac{1}{\pi(1+x^2)}, -\infty < x < \infty$

$$F(x) = P(X \le x)$$

$$= \int_{-\infty}^{x} f(x)dx = \frac{1}{\pi}\int_{-\infty}^{x}\frac{dx}{1+x^2} = \frac{1}{\pi}\left[\tan^{-1}x\right]_{-\infty}^{x} = \frac{1}{\pi}\left(\frac{\pi}{2}+\tan^{-1}x\right)$$

Q.3 A BSC (Binary Symmetric Channel) error probability is P_e. The probability of transmitting '1' is Q, and that of transmitting '0' is $(1 - Q)$ as in figure below. Calculate the probabilities of receiving 1 and 0 at the receiver?



Solution:

If x and y are the transmitted digit and the received digit respectively, then for a BSC,

$$P_{y|x}(0|1) = P_{y|x}(1|0) = P_e$$
$$P_{y|x}(0|0) = P_{y|x}(1|1) = 1 - P_e$$

Also, $\qquad P_x(1) = Q$ and $P_x(0) = 1 - Q$

We have to find, $P_y(1)$ and $P_y(0) = ?$

$\therefore \qquad P_y(1) = P_x(0)\,P_{y|x}(1|0) + P_x(1)\,P_{y|x}(1|1) = (1 - Q)P_e + Q(1 - P_e)$

also, $\qquad P_y(0) = P_x(0)P_{y|x}(0|0) + P_x(1)\,P_{y|x}(0|1) = (1 - Q)(1 - P_e) + QP_e$

Q.4 For the triangular distribution

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and variance.

Solution:

$$\text{Mean} = E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x \cdot x\,dx + \int_1^2 x(2-x)\,dx = \int_0^1 x^2\,dx + \int_1^2 (2x - x^2)\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 + \left[2\left(\frac{x^2}{2}\right) - \frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{3} + \left[\left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)\right] = \frac{1}{3} + \frac{4}{3} - \frac{2}{3} = 1$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^1 x^2 x\,dx + \int_1^2 x^2(2-x)\,dx$$

$$= \int_0^1 x^3\,dx + \int_1^2 (2x^2 - x^3)\,dx = \left[\frac{x^4}{4}\right]_0^1 + \left[2\left(\frac{x^3}{3}\right) - \frac{x^4}{4}\right]_1^2$$

$$= \frac{1}{4} + \left[\left(\frac{16}{3} - \frac{16}{4}\right) - \left(\frac{2}{3} - \frac{1}{4}\right)\right] = \frac{1}{4} + \frac{16}{3} - 4 - \frac{2}{3} + \frac{1}{4} = \frac{7}{6}$$

$$\text{Var}(X) = E(X^2) - E(X)^2 = \frac{7}{6} - (1)^2 = \frac{1}{6}$$

Q.5 The joint density function of two continuous random variables is given by

$$f(x, y) = \begin{cases} xy/8, & 0 < x < 2,\ 1 < y < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find (a) $E(X)$, (b) $E(Y)$ and (c) $E(2X + 2Y)$.

Solution:

(a)
$$E(X) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xf(x, y)\,dx\,dy = \int_{x=0}^{2}\int_{y=1}^{3} x(xy/8)\,dx\,dy = \frac{4}{3}$$

(b)
$$E(Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} yf(x, y)\,dx\,dy = \int_{x=0}^{2}\int_{y=1}^{3} y(xy/8)\,dx\,dy = \frac{13}{6}$$

(c)
$$E(2X + 3Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (2x + 3y)\,dx\,dy = \int_{x=0}^{2}\int_{y=1}^{3} (2x + 3y)(xy/8)\,dx\,dy = \frac{55}{6}$$

Q.6 A WSS random process $x(t)$ is applied to the input of an LTI system with impulse response

$$h(t) = 3e^{-2t}\,u(t)$$

Find the mean value of the output $y(t)$ of the system, if $E[x(t)] = 2$. Here E [·] denotes the expectation operator.

Solution:

The output $y(t)$ is the convolution of the input $x(t)$ and the impulse response $h(t)$.

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t - \tau) \cdot d\tau$$

$$\therefore \quad E[y(t)] = \int_{-\infty}^{\infty} h(\tau) \cdot E[x(t - \tau)] \cdot d\tau$$

$$E[y(t)] = H(0) \times E[x(t)]$$

$$\boxed{E[y(t)] = E[x(t)] \cdot H(0)}$$

where, $H(0) = H(\omega)\big|_{\omega=0}$ and $H(\omega)$ = Fourier transform of $h(t)$

Given $E[x(t)] = 2$, $\qquad h(t) = 3e^{-2t}u(t)$

Taking Fourier transform, $\qquad H(\omega) = \dfrac{3}{2+j\omega} \Rightarrow H(0) = \dfrac{3}{2}$

$$E[y(t)] = 2 \times \frac{3}{2} = 3$$

Q.7 Suppose that two signals $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(0, T)$. A sample function $n(t)$ of a zero-mean white noise process is correlated with $s_1(t)$ and $s_2(t)$ separately, to yield the following variables:

$$n_1 = \int_0^T s_1(t)\, n(t)\, dt \quad \text{and} \quad n_2 = \int_0^T s_2(t)\, n(t)\, dt$$

Prove that n_1 and n_2 are orthogonal.

Solution:

$$E[n_1 n_2] = E\left[\int_0^T s_1(u) n(u) du \int_0^T s_2(v) n(v) dv\right]$$

$$= \int_0^T \int_0^T s_1(u) s_2(v) E[n(u) n(v)]\, du\, dv$$

$n(t)$ is a white noise process.

So, $\qquad R_N(\tau) = \dfrac{N_0}{2}\delta(\tau)$

$$E[n(u)\, n(v)] = \frac{N_0}{2}\delta(u-v)$$

Hence, $\qquad E[n_1 n_2] = \dfrac{N_0}{2}\displaystyle\int_0^T \int_0^T s_1(u) s_2(v) \delta(u-v)\, du\, dv$

$$= \frac{N_0}{2}\int_0^T s_1(u) s_2(u)\, du$$

$= 0 \qquad \because s_1(t)$ and $s_2(t)$ are orthogonal over $(0, T)$

$E[n_1 n_2] = 0$. So, n_1 and n_2 are also orthogonal.

Q.8 Find the time autocorrelation function of the signal $g(t) = e^{-at}\, u(t)$ and from this obtain the energy spectral density (ESD) of $g(t)$.

Solution:

Auto correlation function,

$$R_x(\tau) = \int_{-\infty}^{\infty} g(t)\, g(t-\tau) dt = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-a(t-\tau)} u(t-\tau) dt$$

$$= \int_{\tau}^{\infty} e^{-2at} e^{a\tau} dt = e^{a\tau}\int_{\tau}^{\infty} e^{-2at} dt = \frac{e^{a\tau}}{-2a}\left[e^{-2at}\right]_{\tau}^{\infty} = \frac{e^{a\tau}}{2a} e^{-2a\tau} = \frac{e^{-a\tau}}{2a}$$

Similar process is valid for negative side because for real $g(t)$, $R_x(\tau)$ is even function of time

$$R_x(\tau) = \frac{e^{-a|\tau|}}{2a}$$

Now we know that

$$R_x(\tau) \xrightarrow{F.T} S_x(\omega)$$

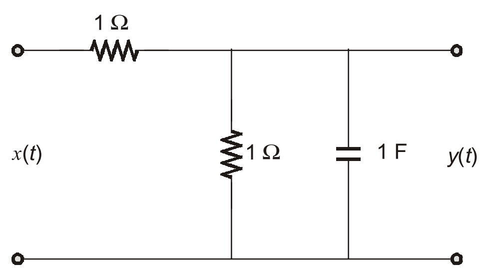
Energy spectral density

$$S_x(\omega) = \frac{1}{\omega^2 + a^2} \triangleq |G(\omega)|^2, \text{ where } G(\omega) = F.T\,[g(t)]$$

Q.9 If the input to a low-pass filter as shown below in Figure is a random process $x(t)$ with autocorrelation function, $R_x(\tau) = 5\delta(t)$, then find

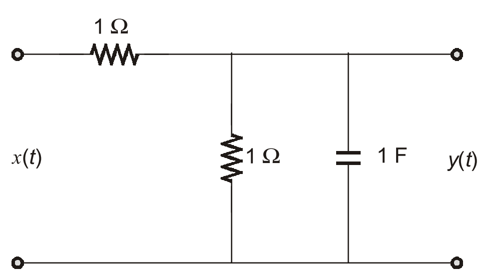
(i) Power spectral density of the output random process;

(ii) Average power of output random process.

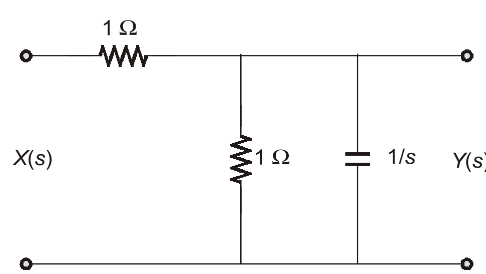


Solution:

Given low pass filter,



By taking Laplace transform



Transfer function, $H(s) = \dfrac{1}{s+2}$; $\quad H(j\omega) = \dfrac{1}{j\omega + 2}$

Input Auto-correlation function

$$R_x(\tau) = 5\delta(t)$$

Power spectral density $S_x(\omega) = F[R_x(\tau)] = 5$

Output power spectral density

$$S_y(\omega) = |H(j\omega)|^2 S_x(\omega)$$

$$S_y(\omega) = \frac{5}{\omega^2 + 4}$$

Output Auto-correlation function = $R_y(\tau) = \dfrac{5}{4}e^{-2|t|}$

$\because \quad e^{-a|\tau|} \xleftrightarrow{\text{CTFT}} \dfrac{2a}{\omega^2 + a^2}$

(ii) Average power of the output process = $R_y(0)$

$$P_y = \frac{5}{4}\text{ W}$$